High Resolution Radar Sensing via Compressive Illumination

Emre Ertin
Lee Potter, Randy Moses, Phil Schniter, Christian Austin, Jason Parker

The Ohio State University

New Frontiers in Imaging and Sensing Workshop
February 17, 2010
Radar: Wide Area-Coverage, all weather, day/night, persistent illumination
High resolution imaging of objects and tracking of moving targets
Wideband Multichannel Radar is a crucial component of research in:

- Emerging Applications in Urban Setting
  - Collaborative Layered Persistent Sensing
  - Thru-wall Surveillance
  - Cognitive Radar Networks
Wideband Multichannel Radar is a crucial component of research in:

- Emerging Applications in Urban Setting
  - Collaborative Layered Persistent Sensing
  - Thru-wall Surveillance
  - Cognitive Radar Networks

- Emerging Signal Processing Techniques
  - Waveform Adaptivity
  - MIMO Radar Systems
  - 3D SAR
  - Radar Diversity Techniques
Software Defined Radar Sensor

Recent advances in high speed A/D and D/A and fast FPGA structures for DSP enabled real time decisions and on the fly waveform adaptation

- **Next Generation Radar Sensors**
  - Software Configurable for multimode operation: Imaging-Tracking
  - Multiple TX/RX chains to support MIMO Radar
  - Independent waveforms for TX and coherent processing in RX

![Diagram of Software Defined Radar Sensor](image-url)
OSU SDR Sensors

1. 1 Tx - 1 Rx Software Defined MicroRadar
   - Software Defined Waveforms
   - FPGA/DSP for online processing
   - Single Channel
   - 125 MHz BW, 5.8 GHz

2. 2 Tx - 4 Rx Software Defined Radar Testbed
   - UWB 7.5 GHz Tx-Rx Bandwidth (0-26 GHz center)
   - Programmable Software Defined Waveforms
   - Fully coherent multichannel operation for MIMO
   - Limited Online Processing, Ideal for Field Measurements

3. 4 Tx - 4 Rx MIMO Software Defined Radar Sensor
   - Programmable Software Defined Waveforms
   - Multiple FPGA/DSP Chains for online processing
   - Fully coherent multichannel operation for MIMO
   - 500 MHz BW frequency agile frontend (2-18 GHz)

www.ece.osu.edu/~ertine/RFtestbed
Emerging applications stretch the resolution and bandwidth capabilities of ADC technology

- COTS ADCs have limited resolution at high sampling rates
- Power consumption quadruples for additional bit of resolution

[R.H. Walden, "Analog-to-Digital Converter Survey and Analysis", IEEE JSAS]
Wideband Radar Technology

Emerging applications stretch the resolution and bandwidth capabilities of ADC technology

- COTS ADCs have limited resolution at high sampling rates
- Power consumption quadruples for additional bit of resolution

[S.R. Walden, "Analog-to-Digital Converter Survey and Analysis", IEEE JSAS]

- Sensing is not just receive processing $y = \Phi \Psi s$ vs $y = \Phi_r \Phi_t \Psi s$
Emerging applications stretch the resolution and bandwidth capabilities of ADC technology

- COTS ADCs have limited resolution at high sampling rates
- Power consumption quadruples for additional bit of resolution

Sensing is not just receive processing \( y = \Phi \Psi s \) vs \( y = \Phi_r \Phi_t \Psi s \)

- Use transmit diversity to shift burden away from ADC
- Transmitter at Radar provides more flexibility
Outline

- Radar Estimation Problem
- Compressive Sensing
- Multifrequency Waveforms for Compressive Radar
- Experimental Results
- Conclusion and Future Work
Radar as Channel Estimation Problem

System Model

\[ y_p = R_p X_p t_p + n_p \quad p = 1 \ldots P \]

- \( y_p \): radar return
- \( X_p \): convolution matrix of the channel response
- \( t_p \): transmit waveform
- \( R_p \): receive processing filter
- \( n_p \): system noise
Radar Sensing Model

System Model

\[ y_p = R_p T_p x_p + n_p \quad p = 1 \ldots P \]

- \( y_p \): radar return
- \( T_p \): convolution matrix of the transmit waveform
- \( x_p \): unknown target response
- \( R_p \): receive processing filter
- \( n_p \): system noise
Radar Sensing Model

System Model

\[ y_p = R_p T_p x_p + n_p \quad p = 1 \ldots P \]
\[ = A(r_p, t_p)x_p + n_p \]

- \( y_p \) : radar return
- \( T_p \) : convolution matrix of the transmit waveform
- \( x_p \) : unknown target response
- \( R_p \) : receive processing filter
- \( n_p \) : system noise
Radar Imaging Problem

Estimate unknown target range profile $x$ from the sampled radar returns $y_p$

$$y_p = A(r_p, t_p)x + n_p$$

- $x$ is sampled at the transmit bandwidth (or higher)
- $y_p$ sampling rate determines ADC requirements
Radar Imaging Problem

Estimate unknown target range profile $x$ from the sampled radar returns $y_p$

$$y_p = A(r_p, t_p)x + n_p$$

- $x$ is sampled at the transmit bandwidth (or higher)
- $y_p$ sampling rate determines ADC requirements

Use prior knowledge about the scene for design of $(r_p, t_p)$ to reduce ADC rate
Example: Stretch Processing

Traditional LFM chirp signals can provide 2-10x sub-Nyquist sampling

- Analog dechirp processing followed by low rate ADCs
- Sample uniformly in frequency; alias to exploit limited swath in range
- Stretch gives compression versus transmit bandwidth

\[ B \frac{\tau}{T_p} \text{ vs } B \]

swath \( \tau = 2R/c \) sec; pulse duration \( T_p \) sec
Outline

- Radar Estimation Problem
- Compressive Sensing
- Multifrequency Waveforms for Compressive Radar
- Experimental Results
- Conclusion and Future Work
Signal recovery from projections

Signal Recovery

Inverse problem of recovering a signal $\mathbf{x} \in \mathbb{C}^N$ from noisy measurements of its linear projections

$$y = \mathbf{A}\mathbf{x} + \mathbf{n} \in \mathbb{C}^M. \quad (1)$$

Focus: $\mathbf{A} \in \mathbb{C}^{M \times N}$ forms a non-complete basis with $M << N$.

Ill posed recovery problem is regularized:

1. the unknown signal $\mathbf{x}$ has at most $K$ non-zero entries
2. the noise process is bounded by $\|\mathbf{n}\|_2 < \epsilon$. 
Sparsity Regularized Inversion

\[ y = Ax + e \]

- data
- Measurement model
- basis
- coefficients
- noise

Sparse: \( x \) has \( K \ll N \) nonzero elements in some known basis
High-frequency scattering center decomposition

3D+ from circular SAR
Gotcha public release
8 passes, HH & VV
Compressive Sensing

Sparsity Regularized Inversion

Sparse Signal Recovery Problem

$$\min_{\mathbf{x}} \|\mathbf{x}\|_0 \text{ subject to } \|\mathbf{Ax} - \mathbf{y}\|_2^2 \leq \epsilon,$$
Sparsity Regularized Inversion

Convex Optimization for Sparse Recovery

\[ \min_{\mathbf{x}} \|\mathbf{x}\|_1 \text{ subject to } \|A\mathbf{x} - \mathbf{y}\|_2^2 \leq \epsilon. \]

Provides a bounded error solution to the NP-complete sparse recovery problem, if \( \delta_{2K}(A) < \sqrt{2} - 1 \)
**Geometric Intuition**

- **Illustration: 3 unknowns, 2 equations**
  \[ \hat{x} = \arg \min_x \|x\|_q \quad \text{s. t.} \quad \|Ax - y\|_2 \leq \epsilon \]

- $q=2$
  Min-norm Least-Squares.
  Not sparse.

- $q=1$
  Convex problem.
  Sparse solution.

- $q \ll 1$
  Not convex.
  For $q=0$, solution is NP hard.
Sparsity Regularized Inversion

Convex Optimization for Sparse Recovery

$$\min_{x} \|x\|_1 \text{ subject to } \|Ax - y\|_2^2 \leq \epsilon.$$ 

Provides a bounded error solution to the NP-complete sparse recovery problem, if \(\delta_{2K}(A) < \sqrt{2} - 1\)
Convex Optimization for Sparse Recovery

\[
\min_{\mathbf{x}} \|\mathbf{x}\|_1 \text{ subject to } \|A\mathbf{x} - \mathbf{y}\|_2^2 \leq \epsilon.
\]

Provides a bounded error solution to the NP-complete sparse recovery problem, if \(\delta_{2K}(A) < \sqrt{2} - 1\)

Restricted Isometry Constant

RIC \((\delta_s)\) for forward operator \(A\) is defined as the smallest \(\delta \in (0, 1)\) such that:

\[
(1 - \delta_s)\|\mathbf{x}\|_2^2 \leq \|A\mathbf{x}\|_2^2 \leq (1 + \delta_s)\|\mathbf{x}\|_2^2
\]

holds for all vectors \(\mathbf{x}\) with at most \(s\) non-zero entries.
Sparsity Regularized Inversion

Convex Optimization for Sparse Recovery

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \text{ subject to } \|A\mathbf{x} - \mathbf{y}\|_2^2 \leq \epsilon.$$ 

Provides a bounded error solution to the NP-complete sparse recovery problem, if $\delta_{2K}(A) < \sqrt{2} - 1$.
Compressive Sensing in Radar Imaging

To account for anisotropic scattering, complex-valued data, sparsity in various domains, use penalty terms adopted in image processing in complex data setting [Cetin, Karl, others 2001]

\[
\text{min } \| y - Ax \|_2^2 + \lambda_1 \| x \|_p^p + \lambda_2 \| D|x| \|_p^p
\]
Compressive Sensing in Radar Imaging

- 3D Imaging: Combine 2D data from few passes to form 3D Imagery. Sparse sampling in elevation leads to high sidelobes in slant-plane height in L2 reconstruction.

Sparsity Regularized Inversion

Mutual Coherence

Mutual coherence of the forward operator $\mathbf{A}$:

$$\mu(\mathbf{A}) = \max_{i \neq j} |\mathbf{A}_i^H \mathbf{A}_j|.$$  \hspace{1cm} (2)

RIC is bounded by $\delta_s < (s - 1)\mu$

Design Transmit Waveforms and Receive Processing to minimize mutual coherence of the forward operator $\mathbf{A}(r_p, t_p)$
Mutual Coherence

Mutual coherence of the forward operator $\mathbf{A}$:

$$\mu(\mathbf{A}) = \max_{i \neq j} |\mathbf{A}_i^H \mathbf{A}_j|.$$  \hspace{1cm} (2)

RIC is bounded by $\delta_s < (s - 1)\mu$

Design Transmit Waveforms and Receive Processing to minimize mutual coherence of the forward operator $\mathbf{A}(r_p, t_p)$

Random waveforms sacrifice stretch processing gain

We consider multifrequency chirp signals
Compressive Sensing

Measurement Kernels in Radar

- Classical radar ambiguity function yields mutual coherence, $\mu$
- RIP constant: $\delta_s < (s - 1)\mu$
- Past history of randomization in radar
  - array geometries
  - pulse repetition jitter
  - noise waveforms
Outline

- Radar Estimation Problem
- Compressive Sensing
- Multifrequency Waveforms for Compressive Radar
- Experimental Results
- Conclusion and Future Work
Multi-frequency Chirp Waveforms

Multi-frequency chirp, $K$ sub-carriers

$$f_p(t) = \sum_{k=1}^{K} e^{j\Phi_p^k \text{rec}(\frac{t}{\tau})} \exp \left( j2\pi(f_k^p t + \frac{\alpha}{2}t^2) \right)$$

Received signal for target at distance $d$ ($t_d = \frac{2d}{c}$)

$$s_p(t) = c \sum_{k=1}^{K} e^{j\Phi(f_k^p, t_d, \phi_k^p) \text{rec}(\frac{t}{\tau} - t_d)} \times \exp \left( j2\pi((f_k^p - f_0 - \alpha t_d) t) \right)$$

$$\phi(f_k^p, t_d, \phi_k^p) = \phi_k^p - 2\pi f_k^p t_d$$
Receive Processing

- Transmit: Illuminate scene with sum of multi-frequency chirps; randomize subcarrier frequencies and phases.
- Receive:
  - Analog: Mix with a single chirp and sample with a slow A/D with wide analog bandwidth to obtain randomized projections.
  - Software: Use compressive sensing recovery algorithm with provable performance guarantees.

For multiple pulses, dual of Xampling [Mishali & Eldar] which uses fixed bank of hardware mixers on receive to alias wideband signal to baseband.
Receive Processing

For multiple pulses:
- target at 5 m
- 500 MHz bandwidth transmission; 5 Msps ADC

Observe:
- low-rate ADC aliases wide-band returns to common baseband
- Subcarrier phases and frequencies yield randomized projections

![Graphs showing range vs. pulse](image)
Outline

- Radar Estimation Problem
- Compressive Sensing
- Multifrequency Waveforms for Compressive Radar
- Experimental Results
- Conclusion and Future Work
Experimental Results

- Basis Pursuit Recovery of Sparse Vector ($K=10$) with 1/5 undersampling at 20dB SNR
Experimental Results

Experimental Results: Single Pulse

- Top row: MSE as a function of SNR & sparsity; bottom row: histogram of $A' \ast A$ magnitudes (coherence)

1 Chirp

7 Chirps

15 Chirps
Experimental Results

- MSE as a function of SNR and Sparsity and Mutual Coherence

SubCarrier=1  Subcarrier=7  Subcarrier=15
Experimental Results

Multiple Channels: Multiple Pulses, Orthogonal Waveforms, Polarization
- MSE as a function of SNR and Sparsity for 15 Subcarriers

Channels=1

Channels=2

Channels=3

Channels=4
Hardware Experiment

- Transmit waveform consists of 11 non-overlapping 50 MHz bandwidth chirps of total approximate bandwidth of 550 MHz.
- Single Pulse of 10 μseconds
- Single stretch processor sampling at a rate of 5 Msample/sec (I/Q)
Experimental Results

Hardware Experiment

- Transmit waveform consists of 11 non-overlapping 50 MHz bandwidth chirps of total approximate bandwidth of 550 MHz.
- Single Pulse of 10 μseconds
- Single stretch processor sampling at a rate of 5 Msample/sec (I/Q)
Hardware Experiment

- Transmit waveform consists of 11 non-overlapping 50 MHz bandwidth chirps of total approximate bandwidth of 550 MHz.
- Single Pulse of 10 μseconds
- Single stretch processor sampling at a rate of 5 Msample/sec (I/Q)
Hardware Experiment

- Transmit waveform consists of 11 non-overlapping 50 MHz bandwidth chirps of total approximate bandwidth of 550 MHz.
- Single Pulse of 10 µseconds
- Single stretch processor sampling at a rate of 5 Msample/sec (I/Q)
Outline

- Radar Estimation Problem
- Compressive Sensing
- Multifrequency Waveforms for Compressive Radar
- Experimental Results
- Conclusion and Future Work
Conclusion and Future Work

- We presented wideband compressive radar sensor based on multifrequency FM waveforms.
- Shift the complexity from the receiver to transmitter.
- Future work in characterizing coherence of the resulting forward operator and sparse construction performance.
- Extend compressive sampling on the other dimensions of the radar data cube.