Geospatial Image Fusion

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Outline
- Image modalities
  - Multispectral (low-dimensional) vs. Hyperspectral (high-dimensional)
- Multispectral image fusion: IHS Transform
- Hyperspectral image fusion: VWP
- Extension to density estimation

Geospatial Data
- Government agencies have to gather geospatial intelligence in many situations: military, environmental, mapping, agricultural, emergency response.
- Data comes in different modalities.
  - Images: Color (Google maps), Infrared, Hyperspectral
  - Image-like data: RADAR, LIDAR, gravimetric
  - Point data: census data, events (e.g. crimes)
  - Text: field reports, news reports, rumors
- Each modality offers different advantages.

Image Modalities
- A multispectral image is typically a 4-6 band image: RGB + one or more infrared bands.
- A hyperspectral image typically has ~200 bands, each band representing the response to a precise wavelength of light.
- So each pixel is a 200-dimensional signal.
- The signal can potentially identify the material present.
Difficulty with spectral images
- It is difficult to build a camera with both high spectral and spatial resolution.
- As the camera sensors are fine-tuned to a specific wavelength of light, the sensor loses spatial accuracy.
- So obtaining the extra image bands comes at the price of "bigger pixels".

Pan-sharpening
- To compensate for the spectral / spatial trade-off, many earth-observing satellites are equipped with 2 types of sensors:
  - Multispectral -- a 4-band image with good spectral (color) information but low spatial detail
  - Panchromatic -- a grayscale (1-band) image with high spatial detail, but no spectral information.
- For example, in the Quickbird satellite the panchromatic image has 0.6m resolution and the multispectral image has 2.4m resolution.

Spectral Response
- To get the higher spatial accuracy the panchromatic sensor has a very wide spectral response.

The Fusion Problem
- A tremendously ill-posed problem.
- 1 big color pixel + 16 small gray pixels = 16 color pixels
The standard pan-sharpening technique is the IHS (Intensity-Hue-Saturation) transform.

For a multispectral image $M$ and a panchromatic image $P$, compute

$$F_i = M_i + P - I$$

$$I = M_1 + M_2 + M_3 + M_4$$

We can generalize the IHS model to arbitrary coefficients.

Ideally, these coefficients would be derived from information about the sensor. (Choi-Cho-Kim, 2008) suggested experimentally determined values for the IKONOS satellite

$$I = 0.1M_1 + 0.25M_2 + 0.0833M_3 + 0.567M_4$$

Without knowing the sensor details, can we reverse engineer the coefficients from the image? We want to approximate the panchromatic image as a linear combination of the multispectral bands:

$$I = \alpha_1M_1 + \alpha_2M_2 + \alpha_3M_3 + \alpha_4M_4 = Pan$$

We calculate the coefficients which minimize the following function:

$$E(\alpha) = \sum_x (\sum_i (\alpha_iM_i(x)) - P(x))^2 + \tau \sum_i (\max(0,-\alpha_i))^2$$

Furthermore, we note that the image colors should match away from edges. If $e(x)$ is an edge detector with $e=0$ away from edges, then we want $F=M$ where $e=0$ and use the standard IHS on the edges.

$$F_i = M_i + e(x)(P - I)$$

$$e(x) = \exp\left(-\frac{\lambda}{\|x\|^2}\right)$$

Other approaches to pan-sharpening
- PCA (Shetigara, 1996)
- Brovey (Brovey, 1992)
- Wavelet Fusion (Zhou-Civico-Silander, 1998)
- P+XS (Ballester-Caselles-Igual-Verdera, 2006)

But all the methods have 3 basic problems
1. Do not guarantee color fidelity
2. Make assumptions on the high res image $P$
3. Do not extend well to higher dimensional images

We proposed a variational (energy) method for pan-sharpening that extends to hyperspectral images.
The Variational Approach

- For example, we might choose to minimize the $H^1$ norm of the image.
  \[ \min_{X} J[X] = \int_{\Omega} |\nabla X| \, dx \]
- This wipes out oscillations in the image.
- But it also wipes out the edges!
- A better choice is the Rudin-Osher-Fatemi Total Variation (TV) norm.
  \[ \min_{X} J[X] = \int_{\Omega} |\nabla X| \, dx \]

Mathematics is used in 2 places.
1. Giving intuition to build a good energy for the problem.
   e.g. TV denoising model (Rudin-Osher-Fatemi, 1992)
   \[ \min_{X} J[X] = \int_{\Omega} |\nabla X| + \lambda \int_{\Omega} (X - X^0)^2 \, dx \]
2. Designing optimization techniques to efficiently minimize the energy.
   e.g. implicit schemes, level sets, operator splitting, graph cuts, Bregman iteration

Now for our problem...

- Given:
  - $P$ - high resolution grayscale image
  - $X^0$ - original low resolution multi-band multispectral image
- Produce:
  A new multi-band image $X$ that fuses the spatial information of $P$ and the spectral information of $X^0$.

Our energy has 3 basic parts...

Part 1: Edge Alignment

- We can’t trust the colors of the panchromatic image $P$, but we can use the edges.
- We need a term that connects our new image $X_n$ to the high-res panchromatic image $P$.
  \[ \nabla \cdot \nabla \times P \]
- The new image $X_n$ should show large change in the $n$ direction, but not in the direction of $t$ (Ballester-Caselles-Verdera-Rouge, 2006).

Part 1: Edge Alignment

- So if we define the panchromatic image’s unit normal to be:
  \[ \theta = \frac{\nabla P}{|\nabla P|} \]
- Then we want our new image $X_n$ to not change across this direction.
  \[ \min_{X_n} \int_{\Omega} |\nabla X_n| \, dx \]
- Integrating by parts gives
  \[ \min_{X_n} \int_{\Omega} |\nabla X_n| + |X_n \cdot \text{div}(\theta)| \, dx \]
Part 2: Wavelets

The wavelet decomposition separates the color and edge information.

A popular pan-sharpening technique is to combine the wavelet coefficients to form a new image. But this mix-and-match approach generally produces strange image artifacts.

We propose matching the high level wavelet coefficients of the images.

\[
\min \sum_{(j,k)} \beta_{j,k} (\gamma_{j,k}^{NS} - \gamma_{j,k}^{PS})^2
\]

where the \( \beta \)'s are the wavelet coefficients of the new image \( X \).

The \( \gamma \)'s are the wavelet coefficients or either the multispectral or panchromatic image.

We propose matching the high level wavelet coefficients of the images.

\[
\text{VWP: Variational Wavelet Pan-sharpening}
\]

Given a multispectral image \( X^S \), we produce a sharpened multispectral image \( X \) by minimizing the energy:

\[
J(X) = \sum_{i=1}^{n} \int_{\Omega} (\nabla X_i \cdot d\sigma(\theta) \cdot X_j) dx + \sum_{(j,k)} \lambda_{j,k} (\gamma_{j,k}^{NS} - \gamma_{j,k}^{PS})^2 + \sum_{j=1}^{n} \int_{\Omega} (X_i - X_j^S) X_j^P | X_j^P|^2 dx + \sum_{j=1}^{n} \int_{\Omega} (X_i - X_j^N) | X_j^N|^2 dx
\]

Force the image \( X \) to respect the edges of the panchromatic whose orientation is given by \( \theta \).

The high level wavelet coefficients should match those of the panchromatic.

Enforce spectral quality by matching to the colors of \( X \) to those in the original data \( X^S \).

A unique solution exists in the space \( BV \) (Moeller-W-Bertozi-Burger, 2012).

Minimization Techniques

We experimented with different techniques

- Gradient descent
- ADI (Peaceman-Rachford, 1955)
- Split Bregman (Goldstein-Osher, 2010)

We proved that the convergence rate of Split Bregman for our problem depends on the number of non-zero gradients in the panchromatic image \( P \).
Bregman Iteration

- The idea of Bregman iteration is from (Yin-Goldfarb-Darbon-Osher, 2008), but it has roots in (Peaceman-Rachford, 1955) and others.
- To minimize the energy

\[ \min_X R(X) + H(X, F) \]

- We iterate, “adding back the noise” each iteration.

\[ X^{n+1} = \arg\min_X [R(X) - R(X^n) - (X - X^n, F^n) + H(X, F^n)] \]

\[ F^{n+1} = F^n - \nabla_X H(X^{n+1}, F) \]

- Several versions for the first minimization: Linearized Bregman, Split Bregman, etc.
- Particularly good when \( R(X) \) is an L1 or TV-like term.

Extension to Hyperspectral Images

- VWP extends nicely to high dimensional hyperspectral images (Moeller-W-Bertozzi, 2009).
  1. The model can handle any number of bands.
  2. The master image does not need to be a panchromatic image. Any high resolution grayscale image will suffice. (We used pictures from Google Maps.)
  3. The model explicitly enforces spectral quality, so the hyperspectral signals will be preserved as much as possible.

Hyper-sharpening

- We examined portions of the 82-band San Diego image.
- Since we didn't have a panchromatic image, we used a high resolution image from Google Maps.
- Images were aligned manually.
Hyper-sharpening

- Features are much more visible in the sharpened image.
- We believe this could potentially improve classification and detection algorithms.

![Hyper-sharpening](image)

Spectral Fidelity

- Away from edges, VWP preserves the original signal.
- On edges, the contrast is changed but the overall signal shape stays the same.

![Spectral Fidelity](image)

Extension to Density Estimation

- The goal of density estimation is to construct the underlying probability density from discrete event data (e.g., crimes, census data, temperatures).
- For event data attached to geographic features, we often visualize density estimates in thematic maps.
- We see these type of images on the news every day.
- Density estimation is more than just visualization.

Kernel Density Estimation

- The standard kernel density estimate can be written as a Maximum Penalized Likelihood Estimation (MPLE) problem.

\[
\hat{u}(x) = \arg\min_{\tilde{u}(x)} \left\{ \int \nabla^2 \tilde{u} \, dx - \mu \sum_{i=1}^{n} \log(u(x_i)) \right\}
\]

where \(u(x)\) is our probability distribution and \(x_i\) are the event locations.

TV MPLE Model

- We propose 2 changes (Smith-Keegan-W-Mohler-Bertozzi, 2010).
  1. The TV norm \(\int |\nabla u| \, dx\) is more appropriate for showing changes in density.
  2. If we can detect "invalid" region \(D\) from geographic data, we can align the density \(u\) with the unit normal to \(D\): \(\hat{u}(x) = \arg\min_{\tilde{u}(x)} \left\{ \int \nabla^2 \tilde{u} \, dx + \lambda \int_{D} \theta \cdot \nabla \tilde{u} \, dx - \mu \sum_{i=1}^{n} \log(u(x_i)) \right\}\)

- Note this term also appears in our VWP model.
That's All Folks!

- Thank you for listening.
- Please send questions or comments to wittmantc@cofc.edu